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# Advancement of offshore wind turbine modelling: fully nonlinear waves on periodic quadratic BEM with corner treatment

STSM final report  
by Agota Mockutė

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COST ACTION TU1304

*WINERCOST*

*Wind energy technology reconsideration to enhance the concept of smart cities*

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# Contents

List of Figures .....	2
1. Mission details .....	3
2. Introduction .....	4
3. Modelling of offshore wind turbines .....	5
3.1. BEM and quadratic elements .....	6
3.2. Corners .....	7
3.3. Periodic BEM .....	8
4. Fully nonlinear periodic sea on rectangular quadratic BEM domain .....	9
4.1. Challenge .....	9
4.2. Implementation .....	10
5. Further advancement and application of the numerical solver .....	11
6. Other benefits of the STSM .....	11
7. Concluding remarks .....	11
References .....	12

## List of Figures

Figure 1. NREL 5MW baseline wind turbine on a monopile support. Courtesy of (Jonkman & Musial 2010). .....	4
Figure 2. Overview of the coupled system and main planned advancements in the hydrodynamic module.....	5
Figure 3. Illustration of the boundary element method on an elliptic domain with 8 quadratic elements. ....	6
Figure 4. Example of corner potential continuity condition applied on a rectangular domain with 1 quadratic element per side. ....	7
Figure 5. Example of imposed periodicity condition on rectangular domain with 1 quadratic element per side. ....	8
Figure 6. Example of domain with 1 quadratic element per side, where both periodic and corner treatment conditions are imposed. ....	9
Figure 7. Example of periodic quadratic BEM domain with 1 element per side and appropriate corner treatment.	10

# 1. Mission details

## 1.1. Participant

The participant of this Short Term Scientific Mission (STSM) is Agota Mockutė, a second-year PhD student on the joint doctoral programme between the University of Florence (UniFI), Italy, and TU Braunschweig (TU-BS), Germany. She is based at UniFI as part of the AEOLUS4FUTURE Innovative Training Network (ITN) under the Marie Skłodowska-Curie Action scholarship.

The participant is supervised by Prof. Claudio Borri (UniFI) and Prof. Klaus Thiele (TU-BS). Acknowledged advice, especially for the stage of the research described in this report, comes from Dr. Enzo Marino (UniFI) and Dr. Claudio Lugni (INSEAN).

## 1.2. Host

This STSM was hosted by Prof. Rüdiger Höffer, the Chair of Windingenieurwesen und Strömungsmechanik, Faculty for Civil and Environmental Engineering, Ruhr-University Bochum (RUB), Germany.

RUB is also a member of the Marie Skłodowska-Curie ITN “AEOLUS4FUTURE - Efficient harvesting of the wind energy” under the European Commission’s Framework Program “Horizon 2020”.

## 1.3. Period

The mission started on 12<sup>th</sup> December 2016 and ended on 28<sup>th</sup> January 2017, excluding a break for winter holidays.

## 1.4. Aim and objectives

The main aim of this mission was obtaining knowledge and network, which would help to advance the numerical solvers used in offshore wind turbine modelling, and then apply them to a wind-wave misalignment study, examining the sensitivity of monopile-supported offshore wind turbines in highly nonlinear waves.

This was achieved through the following:

- Extensive networking
- Familiarisation with local expertise
- Attendance of local lectures, seminars and thesis defences
- Use of locally available resources
- Wind tunnel visits
- Exchange of tools, data and literature
- Consultations with experts of the field

## 2. Introduction

Wind is a clean and inexhaustible source of energy, which provides a high potential to fill in the growing energy demand and ultimately substitute fossil fuels (GWEC 2016). Offshore conditions offer steadier and stronger winds and fewer restrictions concerning noise and visual impact issues, therefore wind turbines can be larger, generating significantly more electricity per turbine (GWEC 2015). However, offshore wind technology is still very costly, and reducing uncertainties in the design process would help to improve the cost-efficiency and reliability of the future designs.

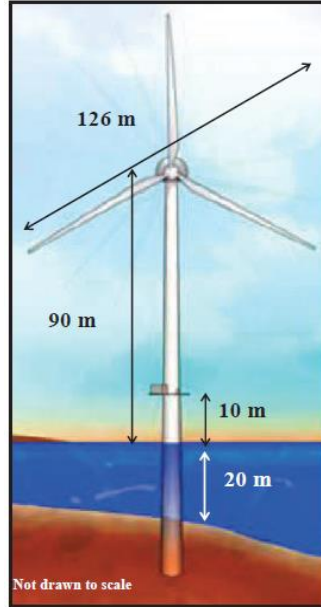


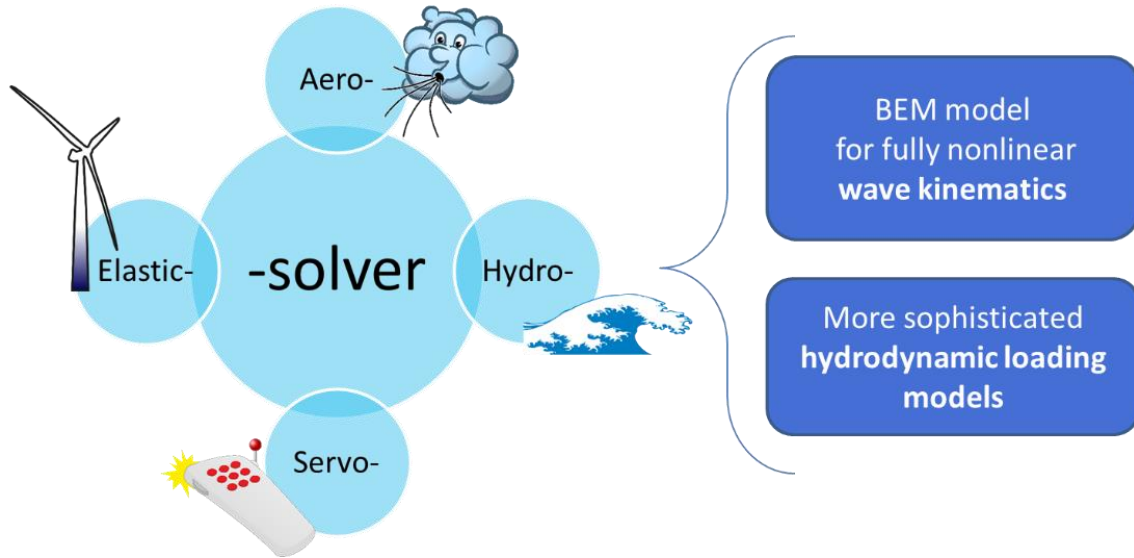
Figure 1. NREL 5MW baseline wind turbine on a monopile support. Courtesy of (Jonkman & Musial 2010).

Offshore wind turbines are commonly placed on monopile support structures, illustrated in Figure 1. When monopile-supported offshore wind turbines are exposed to steep waves, they can exhibit a highly nonlinear resonant amplification response, such as ‘ringing’. Ringing is a nonlinear, non-Gaussian amplification of a response, which reaches high values in few oscillations and decay slowly, therefore portraying a dangerous hazard to the structure (Chaplin et al. 1997; Gurley & Kareem 1998; Grue & Huseby 2002; Schl er et al. 2016). Such excitations have been observed on turbines in parked configuration and have been shown to have significant effects on both extreme and fatigue loads (Marino et al. 2013a; Marino et al. 2015; Marino et al. 2017). In the same environmental conditions where high nonlinear amplifications are seen on parked turbines, the response on operating turbines was negligible, and it was explained by the effect of aerodynamic damping in the direction of the wave loading (Marino et al. 2013b; Marino et al. 2017). However, none of the previous studies have investigated misaligned wind and waves because it is not normally the case of highest loading. Therefore an interest was sparked whether in the case of wind-wave misalignment the nonlinear resonant excitations would be seen on an operating wind turbine too.

Nonetheless, these dangerous nonlinear effects are omitted if linear or weakly nonlinear wave theories are used, which is the case for the majority of current solvers (Robertson et al. 2015; Robertson et al. 2016), therefore a fully nonlinear wave kinematic solver is being advanced as part of this doctoral research project, as discussed in the next section.

### 3. Modelling of offshore wind turbines

The numerical solver, which is verified, used and advanced in this doctoral project work consists of a boundary element method (BEM) model for fully nonlinear wave kinematics, and a hydro-aero-servo-elastic solver, with which the BEM model will be coupled.



**Figure 2. Overview of the coupled system and main planned advancements in the hydrodynamic module.**

Aero-hydro-servo-elastic solvers are composed of four main modules (Jonkman & Buhl Jr. 2005). Aero module is where all the wind data comes in, such as wind speeds and directions, turbulence properties. Servo part allows the user to control the turbine during the simulation; for example, one can set yaw angles, choose fixed or dynamic pitching, set-up emergency shutdown, and so on. Elastic module treats the turbine – its properties and response to the external environmental actions. Hydro part is where the water data is entered, such as wave heights and periods, currents, support structure qualities. It's also the model which chooses which wave kinematics and hydrodynamic loading models should be used.

The choices for wave forcing modelling in the standard solvers is usually limited to linear and weakly nonlinear wave theories, e.g. Airy and Stokes 2<sup>nd</sup> Order, and only Morison's equation for the hydrodynamic loading model. Unfortunately, these have been shown insufficient to capture nonlinear response of the turbine, such as 'ringing' (Robertson et al. 2016). Therefore the BEM model for fully nonlinear gravity waves is being verified and advanced for the PhD work of the participant.

The boundary element method model solves the potential flow equation, initialised by appropriate analytic theories. For fully nonlinear waves Rienecker-Fenton theory is used, therefore imposing no limitations in terms of water depth or wave steepness (Rienecker & Fenton 1981). The solver consists of a rectangular domain bounded by quadratic elements, which is first applied to periodic problems, consequently requiring the combination of three key features: quadratic elements, corner treatment, and periodicity on vertical walls. Each of these three elements is discussed in greater detail in the following subsections.

### 3.1. BEM and quadratic elements

To briefly explain the background, the boundary element method solves for the  $\varphi$  (potential) and  $q$  (flux, derivative of  $\varphi$  in the normal to the surface) values on the boundary  $\Gamma$  which surrounds domain  $\Omega$ , as shown in Figure 3. The domain boundary is discretised into elements, which can be constant, linear, quadratic, cubic, and so on, containing increasingly more nodes, and therefore allowing for more nonlinearity. For example, Figure 3 illustrates an elliptic domain with 8 quadratic elements. Such discretisation allows to solve the problem with Boundary Integral Equations.

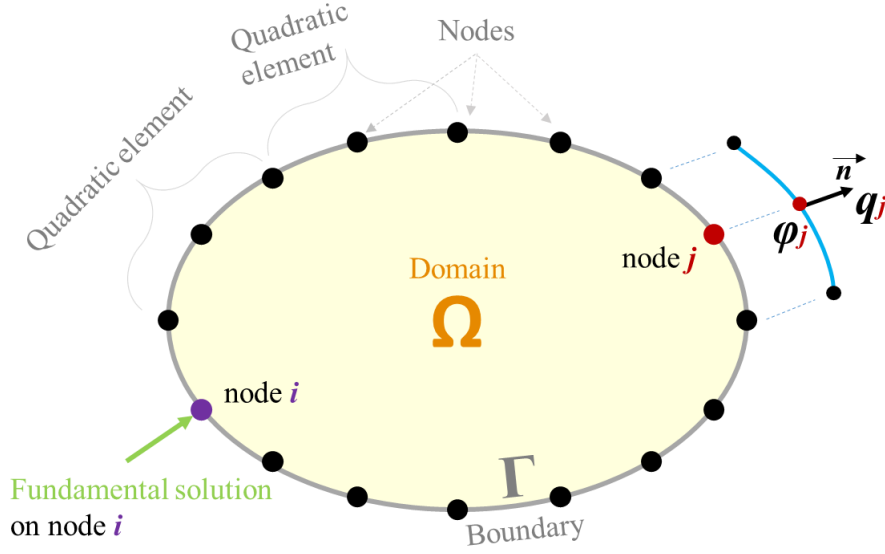


Figure 3. Illustration of the boundary element method on an elliptic domain with 8 quadratic elements.

Fundamental solution is applied to every node in turn, and for each of its positions (e.g. while fundamental solution is applied on node  $i$ ) every element is taken out, and from the coordinates of each node, e.g. node  $j$ , the influence factors  $H$  and  $G$  for potential and flux are computed. Different algorithms are applied if the node  $i$  is within the element of interest in order to avoid singularity, but all details are provided in Brebbia & Dominguez (1998). Therefore by the end, for each node with unique coordinates Eq. 1 is written:

$$H_{ij}\varphi_j = G_{ij}q_j \quad (\text{Eq. 1})$$

The equation system is later solved by redistributing the known (i.e. imposed by the Neumann or Dirichlet boundary conditions) and unknown values to different sides of the equation: collecting all the unknown influence coefficients to matrix  $A$ , all unknown  $\varphi$  and  $q$  values to vector  $X$ , and all the known values (already multiplied by their influence coefficients) to vector  $F$ ; and solving Eq. 2:

$$X = A/F \quad (\text{Eq. 2})$$

Even though the higher order elements come at a cost of a longer simulation time because every node introduces an additional equation to be solved, the quadratic elements are more precise than linear or constant elements, as it has been demonstrated multiple times, for example in Brebbia & Dominguez (1998). Quadratic elements are considered of sufficiently high order to deal with nonlinear problems, such as free water surface of highly nonlinear waves.

### 3.2. Corners

Corners in the BEM domain cause an additional issue, therefore need special treatment. Normally each node has one potential and one flux value; however, a corner has two normal vectors, therefore for a single potential value and a single set of coordinates there are two fluxes. It has been shown that even a small numerical instability at the corners quickly expands throughout the domain (Grilli & Svedsen 1990). Various techniques have been proposed to deal with this problem, such as discontinuous elements (Brebbia & Dominguez 1998), or multiple-flux method (Hague & Swan 2009). Another common and well-validated method is the double-node technique (Grilli et al. 1989). It imposes two nodes with the same coordinates at the corners, which results in equal number of nodes and flux values. For example illustrated in Figure 4, on a square domain with 1 quadratic element per side and double corner nodes, there are 12 nodes for the 12 flux values. However, the four pairs of nodes with identical coordinates lead to four sets of duplicate equations, because there are, as in this example, only 8 nodes with distinctive coordinates which provide unique Eq. 1. Moreover, both of the double corner nodes have a single potential value, and this condition needs to be imposed. Consequently, the method substitutes the spare equations with potential continuity condition, which imposes that the potentials on both corner nodes have to be identical, as illustrated in Figure 4.

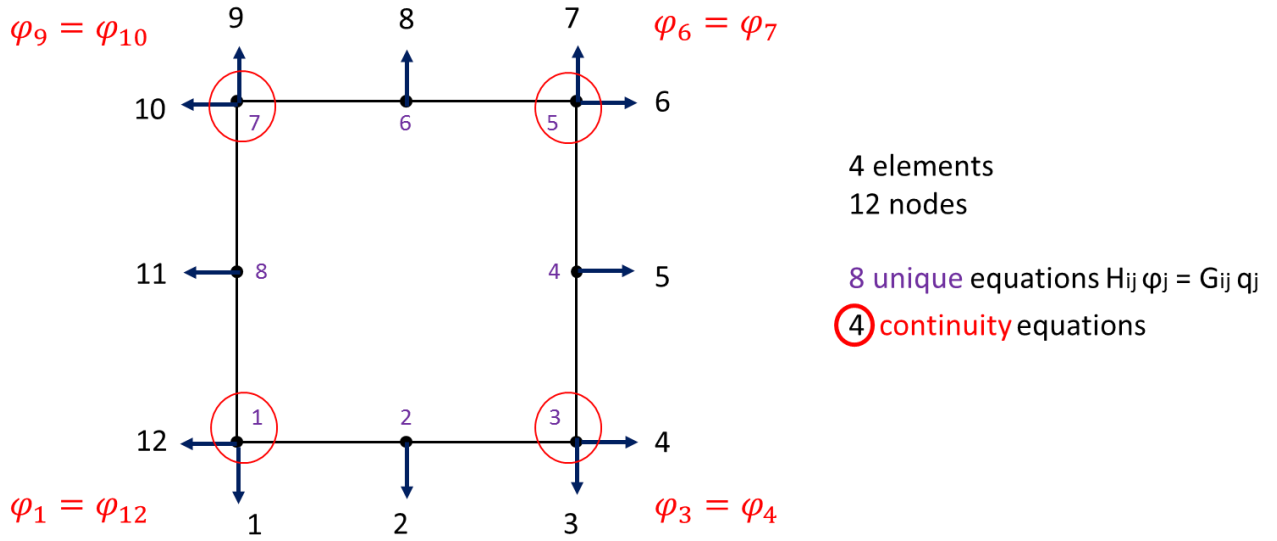


Figure 4. Example of corner potential continuity condition applied on a rectangular domain with 1 quadratic element per side.

Corner-associated issues and their treatment have been a topic of great interest, and to this day numerous guidelines and considerations are provided in the literature. Nonetheless, periodicity is excluded from the discussion, only briefly mentioning that in the case of periodicity corners do not impose an issue (Grilli & Svedsen 1990).

### 3.3. Periodic BEM

BEM is a common choice for periodic gravity wave problems; however, the methodology of imposing periodicity on a rectangular BEM domain is not widely described in detail. Literature was found on simulating periodic waves on a transformed coordinate system, i.e. conformal mapping as first introduced by Longuet-Higgins & Cokelet (1976). However, such method is limited to solely periodic waves and is therefore unsuited for this research project. Studies on physical plane with imposed periodicity were found in literature, e.g. (Vinje & Brevig 1981; Grilli et al. 1989; Ortiz & Douglass 1993), but the methodology was not provided in sufficient detail to be implemented.

Ang (2009) has provided an algorithm for implying a 2D periodicity in a BEM model. It includes imposing boundary conditions on the bottom and free surface as usual, and then considering all variables on the lateral walls as unknowns. This requires as many new equations as there are nodes on the vertical walls, therefore periodicity equations are introduced as illustrated in Figure 5: the potentials at the mirroring points of lateral walls are imposed to be equal, and the fluxes to be of identical magnitude but opposite sign due to the opposite direction of the outward normal vector.

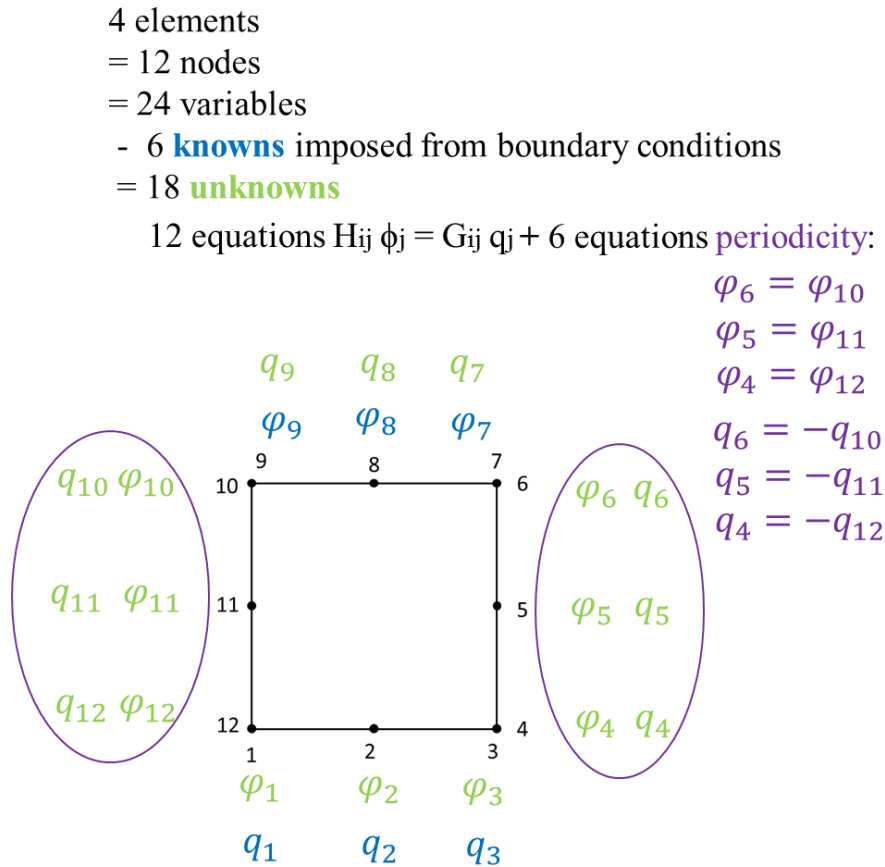


Figure 5. Example of imposed periodicity condition on rectangular domain with 1 quadratic element per side.

However, if only the periodicity condition is imposed, four of the 12 equations are identical due to the corner nodes having identical coordinates (as already explained in Subsection 3.2), and the system cannot be solved. Therefore it has to be used together with the continuity condition. Unfortunately, the methodology of periodic BEM with corner condition on quadratic elements could not be found in the literature and proved to be incompatible if applied directly, as discussed in the next chapter.



## 4. Fully nonlinear periodic sea on rectangular quadratic BEM domain

### 4.1. Challenge

Problem arises when combining the periodic condition with potential continuity in corners, especially on quadratic elements.

Initially all items were accounted for as they were originally intended – using the quadratic elements, imposing the periodicity condition on all variables belonging to the lateral walls, and using the potential continuity on four corners, as illustrated in the Figure 6. The imposed conditions were working as expected: the potentials on the corners and on the lateral walls were equal, the fluxes on the vertical walls were equal in magnitude but opposite in sign. However, the magnitudes of the values were misbehaving, causing numerical instabilities, and after multiple checks it was acknowledged that the system has become over-imposed.

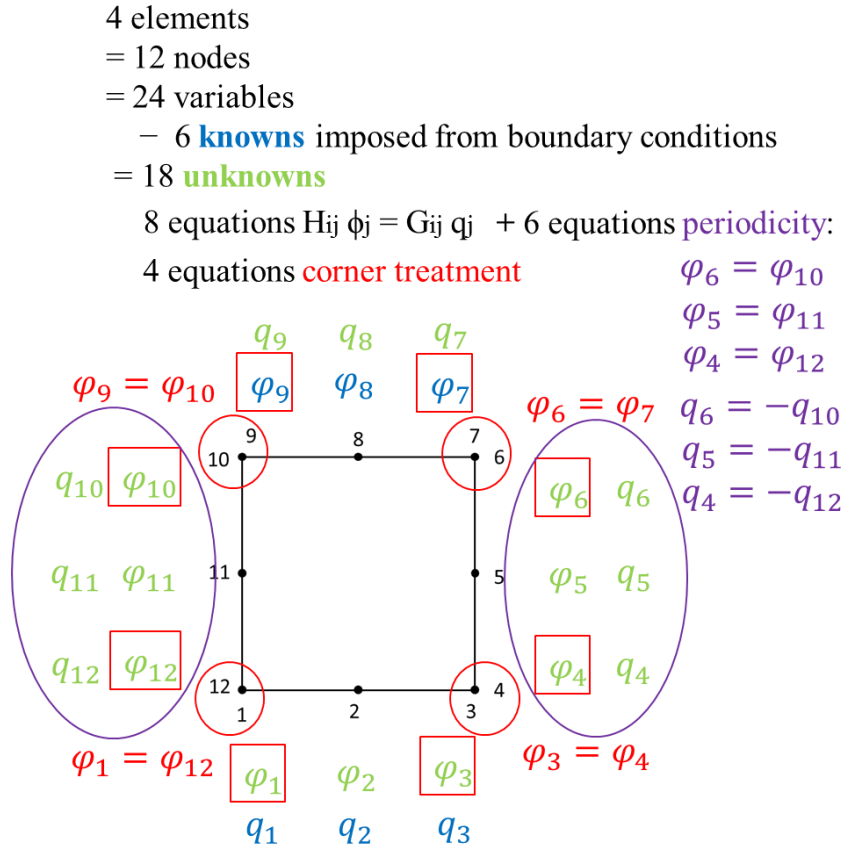


Figure 6. Example of domain with 1 quadratic element per side, where both periodic and corner treatment conditions are imposed.

Numerous unsuccessful trials were attempted to rearrange the system to appropriately define it and avoid over-imposing. They included multiple flux method even though it is not suited for Dirichlet-Dirichlet corners; modelling with no vertical walls at all; other changes in the influence coefficient matrices; imposing Neumann condition on both vertical walls with flux values from analytic solutions even though the values are actually unknown. In the end, a solution which fits the physics behind the model and is properly working was found.

## 4.2. Implementation

The solution to prove successfully functioning was the one which incorporates the continuity condition and periodicity, but not precisely as each of the methods would be implemented separately.

*blue – known values (imposed BC)*

*green – unknowns*

4 elements

12 nodes

8 knowns

16 unknowns

8 unique equations  $H_{ij} \varphi_j = G_{ij} q_j$

2 equations continuity:

$$\varphi_1 = \varphi_{12}$$

$$\varphi_3 = \varphi_4$$

6 equations periodicity:

$$\varphi_5 = \varphi_{11}$$

$$\varphi_4 = \varphi_{12}$$

$$q_6 = -q_{10}$$

$$q_5 = -q_{11}$$

$$q_4 = -q_{12}$$

$$q_7 = q_9$$

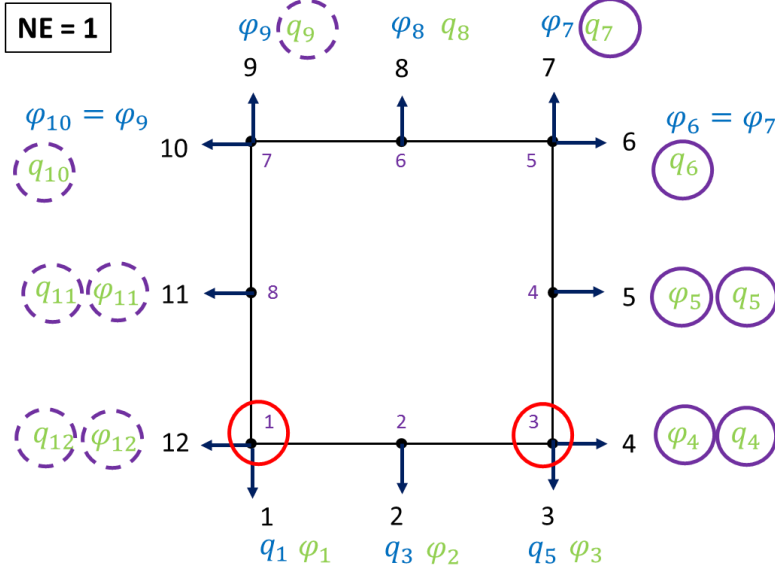


Figure 7. Example of periodic quadratic BEM domain with 1 element per side and appropriate corner treatment.

The main modification is that the potentials at the top corners on lateral walls (e.g.  $\varphi_6$  and  $\varphi_{10}$  in Figure 7) are treated as known values. Due to the potential continuity on corners they are equal to the end values at the free surface, which are in turn imposed from the boundary conditions (BC). This removes two unknowns, but also three equations associated with them: two continuity and one periodicity (for comparison refer to Figure 6). This created a need for an extra equation. After multiple trials of rearranging the system to either remove another unknown or impose another equation, a properly functioning solution was identified: to impose the fluxes at the end of the free surface ( $q_7$  and  $q_9$  in Figure 7) as equal due to periodicity. No such amendment was needed for the bottom corners because there both potentials are unknown as a result of the imposed Neumann boundary condition.

This solution for periodic waves on a rectangular domain proved to be well defined, quickly converging and successfully working, therefore it was used in the further development of the BEM model for fully nonlinear gravity waves.

## 5. Further advancement and application of the numerical solver

Once the regular wave kinematics on the boundary and at the internal points are fully validated, a comparison study of hydrodynamic loading models will be conducted in order to assess which is most suited to deal with high nonlinearities. When both the wave kinematics and forcing modelling are satisfactory, the wind-wave misalignment study on the sensitivity to wave nonlinearities will be conducted.

Next phase involves advancing the model with sloping bed capacity, in order to better represent the natural environment of monopile-supported offshore wind turbines and investigate the nonlinearities induced by shoaling waves.

## 6. Other benefits of the STSM

My professional development was boosted by the expansion of the scientific network during my stay in RUB. The advice I have received from the colleagues at RUB has already helped me approach the problems I have encountered, while the new and strengthened connections will benefit me throughout the rest of my career.

Immense academic help was offered by Prof. Rüdiger Höffer himself and the members of his team by finding time to meet me and exchange knowledge. For example, during my stay in Bochum I was granted access to the FINO offshore site statistics, which are going to be crucial during the systematic wind-wave misalignment impact study. I also received access to all the facilities and internal libraries at Ruhr-University Bochum, have greatly widened my general knowledge on wind engineering related topics during the seminars, lectures and thesis defences of the local students, and the visits to the wind tunnel facility have taught me significant lessons.

Additionally, being based in Bochum gave the perfect opportunity to visit LUH university library in Hanover, which had a copy of a doctoral thesis which was of high interest to the specific part of research but hardly available since there are no digital copies of it.

Finally, on a personal level I also improved my German language skills and familiarised myself more with the working and living culture in Germany.

## 7. Concluding remarks

The STSM has been of great importance for the development of the scientific career for the participant, both in the actual current scientific progress and in the expanded network for future collaborations. Firstly, on the scientific side of the doctoral project the STSM helped to find a way to overcome a present numerical modelling issue that was not well documented in accessible past literature. This opened the way for faster progress in the development and application of the numerical model. Moreover, the STSM was incredibly useful in terms of expanding professional network and exchange of experience, tools and data.

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